

# Power performance of wind turbines over consecutive hills

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## SUMMARY:

High-fidelity large-eddy simulation (LES) is used to investigate power performance of wind turbines over consecutive hills. Wind turbines are parameterized as actuator disk model and consecutive hills are modeled by immersed boundary method in the pseudo-spectral LES solver. The turbines are placed on the crests of five consecutive hills with different slope gradients and their power output and power fluctuations are evaluated. The terrain model of two consecutive hills used in the wind-tunnel research (Karim et al., 2021) is chosen to validate the solver. It is shown that the flow characteristics (including mean streamwise velocity and streamwise turbulence intensity) over consecutive hills in wind tunnel measurements can be reproduced well by our LES framework. Besides, we find that the power output and the power fluctuations in each row are found to be larger as the slope of consecutive hills increases, which is related to the speed-up effect together with faster wake recovery and the higher turbulence intensity, respectively.

*Keywords: LES, Consecutive hills, Power performance.*

## 1. INTRODUCTION

Hilly terrain becomes increasingly appealing from the wind energy industry's point of view due to the fact that the best locations (usually onshore and near the coastline) are becoming occupied and there are no better alternatives. Although there exists considerable literature on atmospheric-boundary layer (ABL) flows over topography, the combination of wind turbines and topography still has much room for investigation (Porté-Agel et al., 2020). Zhang et al. (2022) performed LES to investigate wind-turbine wakes over a two-dimensional hill, suggesting hilltop is the optimal location for turbine placement because the turbine harvests more wind energy due to the speed-up effect and suffers less fatigue loading due to the lower turbulence levels. In contrast to a single mountain, continuous hilly terrain is more common in reality to make up a complex terrain and raises increasing attention in the wind energy community. Hyvärinen et al. (2018) conducted experimental study about the effect of consecutive hilly terrain on wind-turbine performance, observing a faster wake recovery of turbine wakes which may increase the overall power output of turbine arrays. However, previous studies are limited to a single turbine over a single hill or consecutive hills and the combined effect of consecutive hills and wake interaction on the power performance has not been examined. Our research aims at evaluating the power performance of an array of five turbines over consecutive hills.

## 2. NUMERICAL METHODS

### 2.1. LES methodology

The pseudo-spectral solver (LESGO) is used to solve the filtered Navier-Stokes equations in the high-Reynolds number limit on a Cartesian mesh:

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}^d}{\partial x_j} + f_i + \delta_{i1} F_p, \quad (2)$$

where the tilde denotes a spatial filtering at scale  $\tilde{\Delta}$ ,  $t$  is time,  $\tilde{u}_i$  is the instantaneous resolved velocity with subscript  $i = 1, 2, 3$  corresponding to streamwise ( $x$ ), spanwise ( $y$ ), and vertical ( $z$ ) directions, respectively.  $\tilde{p}^*$  is the modified filtered pressure equal to  $\tilde{p}/\rho + \frac{1}{3}\tau_{kk}$ . The force  $f_i$  is added to model the presence of wind turbines and topography.  $\delta_{ij}$  is the Kronecker delta and  $F_p = u_*^2/L_z$  denotes the constant pressure gradient that drives the ABL flows. The subgrid scales are modeled by the Smagorinsky model ( $C_{s0}$  is set to 0.16) with the Mason wall damping model. The wind turbine is parameterized by filter actuator disk model and hilly terrain by immersed boundary method. The thrust force and the power output of a turbine can be formulated as:

$$F_t = -\frac{1}{2}\rho C'_T A_D (u_d^T)^2, \quad P = -F_t u_d^T, \quad (3)$$

where  $A_D$  is the rotor swept area,  $u_d^T$  is disk-averaged and time-filtered velocity and  $C'_T$  is local thrust coefficient ( $C'_T$  is set to 1.33). For the detailed implementation of the solver, interested readers are referred to our recent work (Zhang et al., 2022). For the sake of simplicity hereafter the tilde is omitted.  $(\cdot)$  and  $\sigma_{(\cdot)}$  represent mean value and standard deviation, respectively.

### 2.2. Case settings

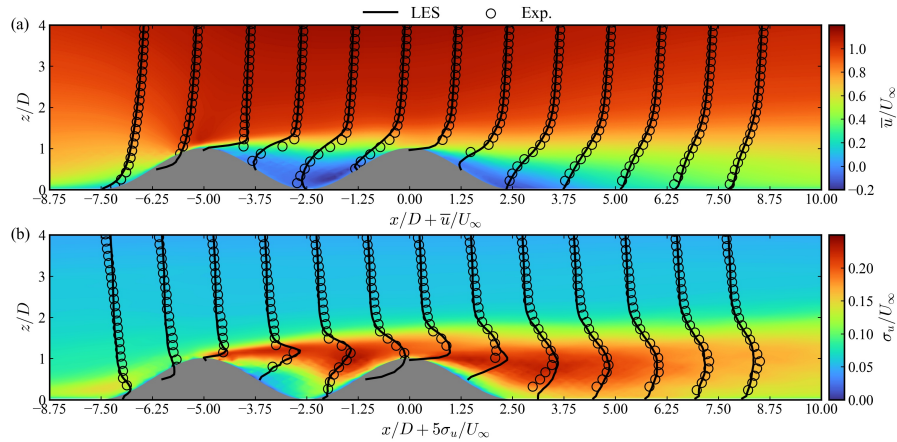
The terrain model of two consecutive hills used in the wind-tunnel research (Karim et al., 2021) is chosen to validate the LES solver. The hill shape is widely used in wind engineering research, which can be expressed by:

$$z_H(x) = H \cos^2\left(\frac{\pi x}{2L}\right), \quad (4)$$

where  $H = 0.06$  m and  $L = 0.15$  m are the height and the half-width of the hill, respectively. To evaluate the combined effect of consecutive hills and wake interaction, the terrain model is extended to five consecutive hills with five turbines sited on the hill crests. The hill height is adjusted to  $0.5H$  to characterize the shape of gentle hills. For turbine dimensions, the diameter ( $D$ ) and the hub height are both set to  $H$ . We use a computational domain of  $L_x \times L_y \times L_z = 45D \times 7D \times 7D$  with a grid resolution of  $N_x \times N_y \times N_z = 288 \times 76 \times 121$ . The chosen resolution of grid points is well within the range that has shown grid independence for simulations of flows over 2D hills and turbine wakes in our previous study (Zhang et al., 2022).

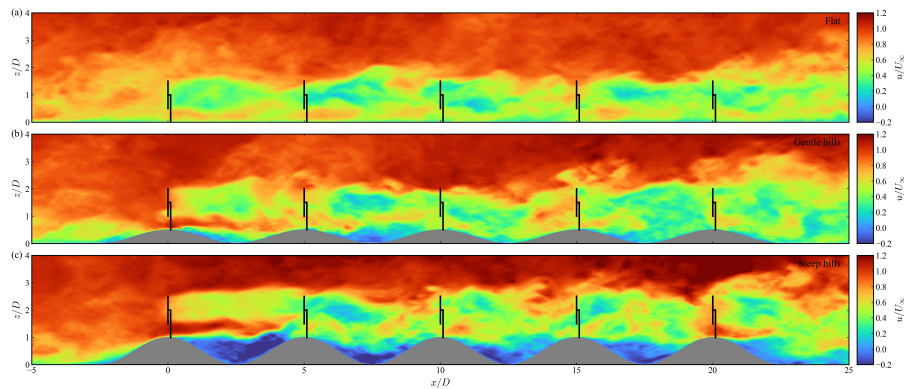
### 3. RESULTS AND DISCUSSION

Fig. 1 shows the contours of the vertical profiles of the mean streamwise velocity as well as the streamwise turbulence intensity in the vertical midplane of the domain for two consecutive steep hills. It is seen that the LES results show overall good agreement with the experimental data at all locations, which validates the capacity of our LES for ABL flows over hills.



**Figure 1.** Contours and vertical profiles of (a) the streamwise velocity and (b) the streamwise turbulence intensity in the vertical midplane of the domain for two consecutive hills (wind tunnel measurements obtained from Karim et al. (2021)).

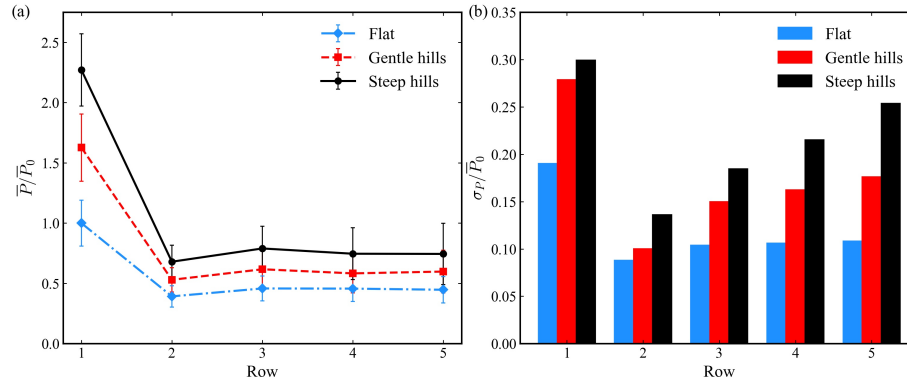
Fig. 2 demonstrates the turbine-wake flows over flat and consecutive hills. Fig. 3 compares the



**Figure 2.** Contours of the instantaneous velocity for turbines over flat and five consecutive hills.

power performance of wind turbines over flat and consecutive hills. The results are normalized by the power output of a stand-alone turbine over flat ( $\bar{P}_0$ ). In Fig. 3 (a), it is seen that more power is generated by the first-row turbine with increasing slope of the hills due to the speed-up effect, reaching 1.6 and 2.3 in the cases of the gentle hills and the steep hills, respectively. However, the power output sharply decreases in the second row, with the most significant drop found in the case of steep hills, which can be understood by the fact that the local thrust coefficients of turbines are set to the same value in the simulations and therefore larger velocity deficit is induced with stronger incoming winds. The power still maintains a higher value in the hill cases compared to the flat case, which is likely related to the faster wake recovery over consecutive hills. Regarding

power fluctuations, they exhibit some similar trends as observed in the mean power. In addition, the power fluctuations become steady from the third row over flat, whereas they continue to increase in the hill cases. It is observed that the power fluctuations in each row also get higher with an increase of the slope, which is attributed to the higher turbulence intensity over steeper consecutive hills.



**Figure 3.** Mean value and standard deviation of power of wind turbines over flat and five consecutive hills.

#### 4. CONCLUSIONS

In our study, we use LES to investigate power performance of wind turbines over consecutive hills. The following conclusions can be drawn from the present study:

- (1) The flow characteristics (including mean streamwise velocity and streamwise turbulence intensity) over consecutive hills in wind tunnel measurements can be reproduced well by our LES framework.
- (2) The power output and the power fluctuations in each row are both found to be higher as the slope of consecutive hills increases, which is related to the speed-up effect together with faster wake recovery and the more intense turbulence intensity, respectively.

#### ACKNOWLEDGEMENTS

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